Calculated Performance of a Dissolved—gas-drive Reservoir by a Phase-behavior Method

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A phase-behavior approach to the prediction of the performance characteristics of a dissolved-gas-drive reservoir is unique in that the problem of choosing flash, differential, or composite-solution gas-oil ratios and formation-volume factors has been circumvented. Data required are a compositional analysis of the reservoir fluid, the bubble point of this fluid, and the relative-permeability curves for the reservoir rock.

Gas-oil ratios and formation-volume factors were calculated under conditions duplicating the performance of the reservoir. A comparison was then made between these results and those obtained by calculations involving a differential, a flash, and a composite process. A vital factor in the solution of the problem is the accuracy of the calculated equilibrium constant. Agreement within 3% was obtained when a calculated differential formation-volume curve was compared with an experimentally determined curve.

Many reservoir engineering calculations depend for their solution on laboratory pressure, volume, and temperature measurements of formation-volume factors and solution gas-oil ratios. How to determine these factors experimentally for the best duplication of the conditions under which reservoir oil is produced to the stock tank has been a matter of disagreement and much speculation in the oil industry.

Three methods are in general use today: *the differential process*, which assumes that as the pressure on an oil is lowered, the gas being liberated from the oil phase is continuously removed from contact with it; *the flash process*, which assumes that all the gas being liberated from the oil phase remains in contact with it; and *the composite process*, which assumes that a differential process occurs in the reservoir and a flash process occurs upon the liquid portion being produced to the stock tank. Each laboratory process has decided limitations and does not represent accurately conditions under which reservoir oil is produced.

A fourth method, a calculative process duplicating the performance characteristics of a dissolved-gas-drive reservoir, is presented here. This process largely escapes the deviations from actual field performance of the other three methods. It may be used to predict recoveries from dissolved-gas-drive reservoirs and may also serve as a tool for the comparison of present methods of determining formation-volume factor and solution gas-oil ratios with the values calculated from any process which duplicates the conditions of either the reservoir or production to the stock tank.

THEORY

Development of Equation

In the hypothetical reservoir chosen for this problem, the oil is assumed to be at its bubble point at original conditions of temperature and pressure. Upon pressure depletion the relative permeabilities of the oil and gas phases are assumed to follow the curves illustrated in Figure 1.

Over an infinitesimal pressure drop a certain number of moles of fluid produced. The number of moles produced from the liquid phase for each component are given by the following equation:

$$x_n \Delta L_t = \Delta L_n \qquad (1)$$

and from the vapor phase,

$$y_n \Delta V_t = \Delta V_n \qquad (2)$$

The moles of component n remaining in the reservoir are then

$$C_{n_1} - \Delta L_n - \Delta V_n = C_{n_2} \qquad (3)$$

The number of moles remaining over a pressure drop $P_2 - P_1$ may be expressed as

$$C_{n_1} - \int_{P_1}^{P_2} x_n \, dL_t - \int_{P_1}^{P_2} y_n \, dV_t - C_{n_2} \qquad (4)$$

Since the variables x_n and y_n cannot be expressed in terms of L_t and V_t, respectively, Equation (4) has no direct solution but must be solved by a trial-and-error procedure. Over small pressure increments Equation (4) may be expressed in the form

$$C_{n_1} - \tfrac{1}{2}[x_{n_1} + x_{n_2}]\Delta L_t - \tfrac{1}{2}[y_{n_1} + y_{n_2}]\Delta V_t = C_{n_2} \qquad (5)$$

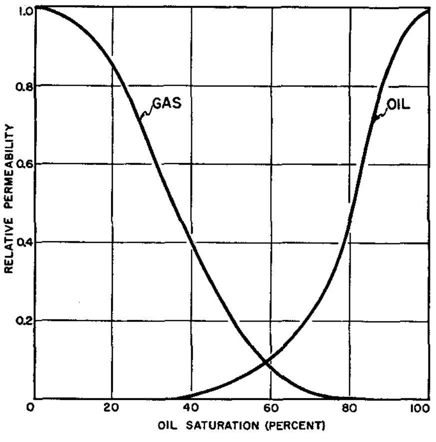

Fig. 1. Relative permeability.

Equation (5) has a direct solution. The mole fractions of each component in the liquid phase and in the vapor phase are assumed at a pressure (P_2), averaged with the known mole fractions in the respective phases at (P_1), and multiplied by the assumed number of moles produced from the liquid or vapor phase. To check the assumptions made, however, three conditions must be satisfied. First, at P_2 the conditions of equilibrium equation (6) must be met:

$$\sum y_{n(P_2)} = \sum \left(\frac{K_n C_n}{(L/V + K_n)V} \right)_{P_2} = 1 \qquad (6)$$

The actual mole fractions of the liquid and vapor phases may be calculated and checked against the assumed mole fractions in Equation (5). Second, the total number of moles of vapor and liquid remaining in reservoir are obtained also from Equation (6). Conversion of these moles to a volume at reservoir conditions by the use of empirical correlations (*2*) will result in a check upon the total number of moles withdrawn. If the volume calculated at P_2 is equal to the original reservoir volume calculated for 100 moles of liquid (in this case), then the total mole withdrawal ($\Delta L_t + \Delta V_t$) was estimated correctly. The third check to be made is for the assumed ratio $\Delta V_t / \Delta L_t$. From the assumed $\Delta V_t / \Delta L_t$ and the mole fraction of the produced fluids, the volumes of gas flowing per volume of liquid at reservoir conditions may be calculated from Equation (7):

$$\frac{\bar{V}_g}{\bar{V}_0} = \frac{ZRT}{P_{(avg)}} \frac{\rho_0}{M_0} \frac{\Delta V_t}{\Delta L_t} \qquad (7)$$

This assumed $\bar{V}_g/\bar{V}_0$ is compared with the average $\bar{V}_g/\bar{V}_0$ calculated from the two instantaneous formations $(\bar{V}_g/\bar{V}_0)_{(i)}$ associated with the liquid saturations at P_2 and P_1. This latter value is calculated by the use of Equation (8).

$$\frac{\bar{V}_g}{\bar{V}_0} = \left(\frac{\mu_0}{\mu_g}\right)_i \left(\frac{k_g}{k_0}\right)_i \qquad (8)$$

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If the assumed values of the mole fractions of each component, the total mole withdrawals, and the relative volume ratio of vapor to liquid withdrawals at reservoir conditions are in agreement with the corresponding calculated values, then Equation (5) is satisfied and the next pressure drop may be attacked.

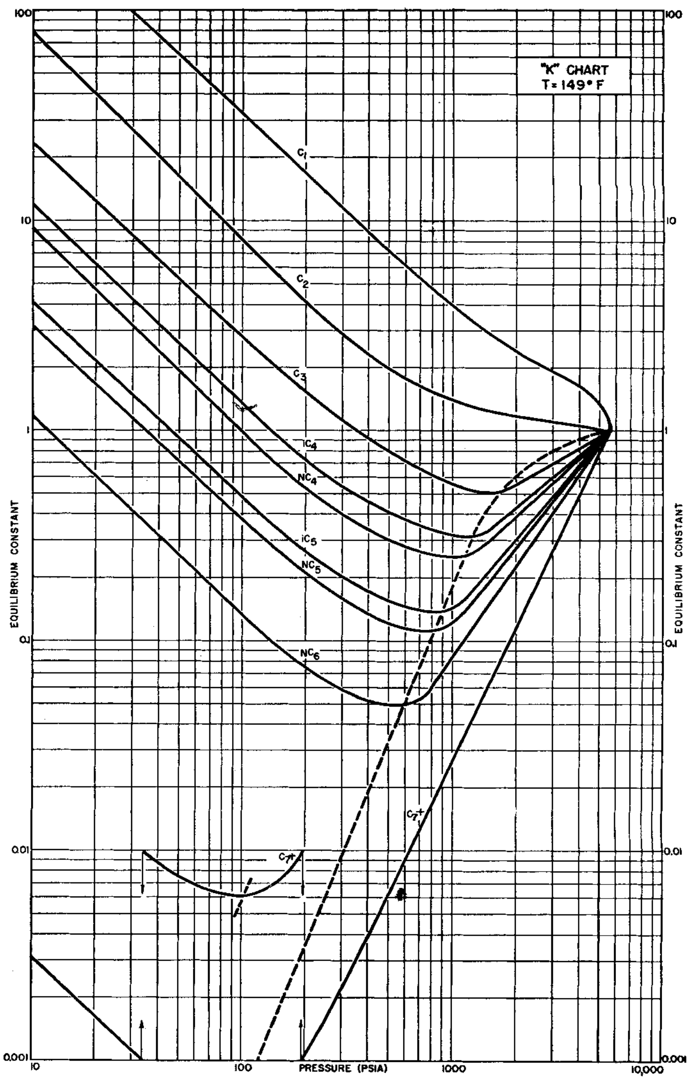

Fig. 2. Equilibrium constants.

PROCEDURE

Calculation of the Performance of a Dissolved-gas-drive Reservoir: Phase-behavior Method

Initial Data. The initial composition of an oil sample (*5*) for which a differential-formation–volume-factor curve was experimentally obtained was determined to be as follows:

Component	Mole fraction
Methane	0.3290
Ethane	0.1489
Propane	0.0393
Isobutane	0.0092
n-Butane	0.0283
Isopentane	0.0108
n-Pentane	0.0171
n-Hexane	0.0358
Heptanes plus	0.3816
	1.0000

Heptanes plus	Saturation pressure
Molecular wt.-217	2,575 lb./sq. in. abs.
Sp. gr.-0.84	at 149°F.

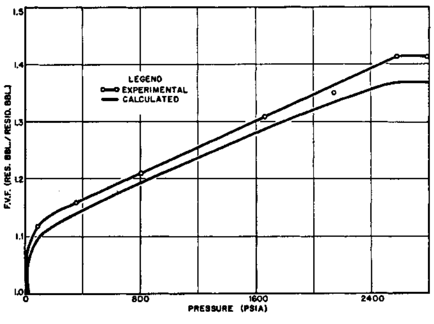

Fig. 3. Differential formation-volume factors.

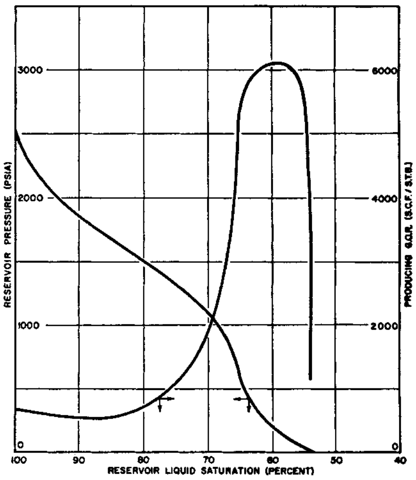

Fig. 4. Calculated pressure and gas-oil-ratio history.

Computed Data. Equilibrium constants, needed for the complete pressure range from the saturation pressure to atmospheric pressure, were calculated by the following sequence of steps:

1. The critical temperature of 810°F., the critical pressure of 265 lb./sq. in. abs., and the molal average boiling point of 502°F. of the heptanes plus (C_7+) fraction were determined by the methods recommended by Brown et al. (*2*).
2. The convergence pressure, 5,500 lb./sq. in. abs., was determined by Hadden's method (*3*).
3. The minimum K values shown in Figure 2 were computed by the procedure presented by G. G. Brown (*2*).
4. The equilibrium vaporization constants for methane shown in Figure 2 were obtained from the tables published by Hanson (*4*), and the constants for the constituents ethane through normal hexane were obtained from Brown's charts (*2*).
5. The values for the C_7+ fraction were obtained when the values for ethane through normal hexane were plotted against their respective normal boiling points and the molal average boiling point of the C_7+ fraction was extrapolated from this curve.
6. By the use of these values and the general shape of equilibrium-vaporization-constant curves for known systems, the curves were extended to the convergence pressure. A final adjustment in the curves made it possible for the computing, through equilibrium-vaporization constants, of a bubble point at 2,575 lb./sq. in abs. by use of the equation

$$\sum (K_n C_n) = 1 \qquad (9)$$

7. The volumes of the liquid and the vapors, both within the reservoir and at stock-tank conditions, were computed by the procedures of Brown (*2*).
8. Viscosity of reservoir oil was calculated by Beal's (*1*) correlation charts, and gas viscosity was obtained from the correlation charts of Muskat (*6*).
9. Relative permeabilities to each phase were determined from Figure 1, a hypothetical relative-permeability graph chosen for this problem.

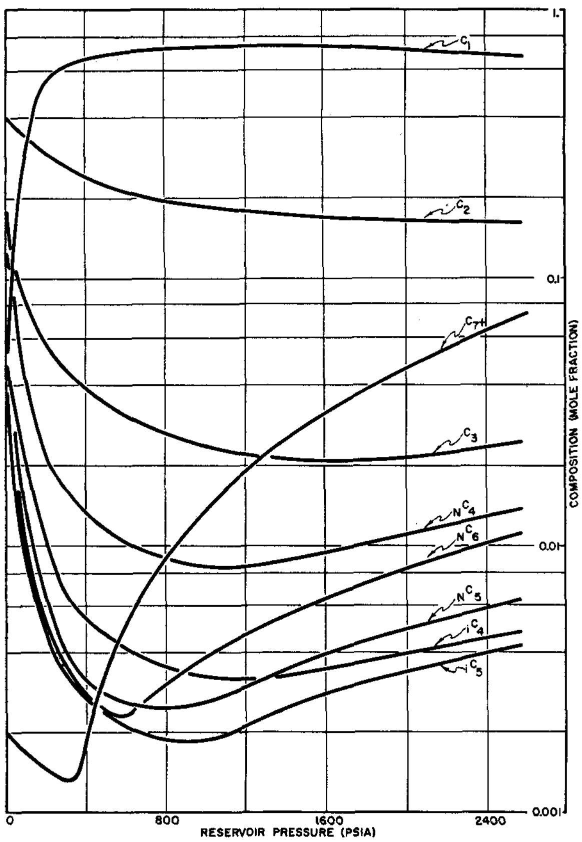

Fig. 5. Calculated reservoir-gas composition.

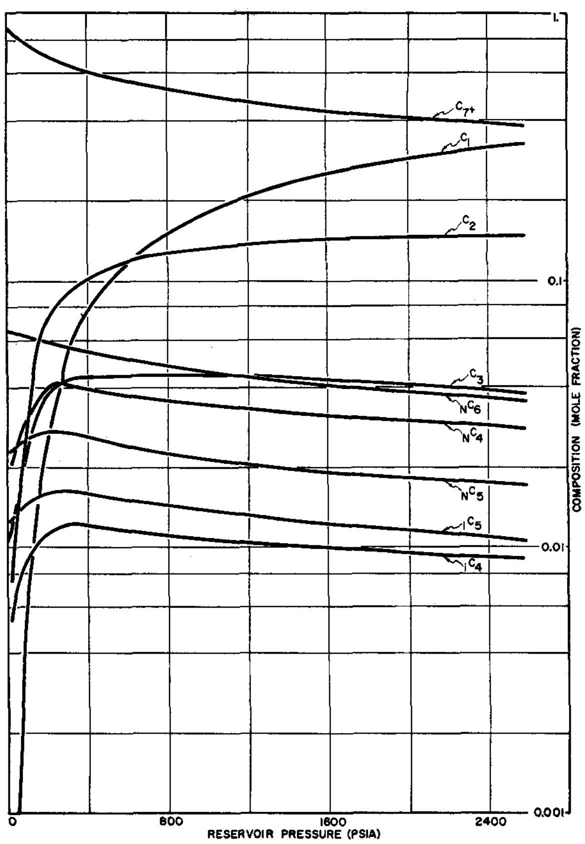

Fig. 6. Calculated reservoir-liquid composition.

The equations presented in the section on theory were solved for the pressure steps of 2,575, 2,000, 1,700, 1,400, 1,100, 700, 300, and 14.7 lb./sq. in. abs. by use of the procedures indicated above. Brinkman's dissertation (7) presents the calculations in tabular form for each of the pressure steps. It must be understood that the time required to make these involved calculations can be considerably shortened by the use of digital computing machines.

Calculation of Formation-volume Factors and Solution Gas-oil Ratios Characteristic of the Three Laboratory Processes

Differential Vaporization Process. The development of the equation

$$Wx_n - y_n\,dW = (W - dW)(x_n - dx) \quad (10)$$

which mathematically duplicates the differential vaporization process follows.

Simplifying Equation (10) and substituting $K_n x_n$ for y_n, one derives the equation

$$W\,dx = K_n x_n\,dW - x_n\,dW \quad (11)$$

Separating variables and integrating results in the equation

$$\int_{x_{n_2}}^{x_{n_1}} \frac{dx}{x_n(K_n - 1)} = \int_{W_2}^{W_1} \frac{dW}{W} \quad (12)$$

If the equilibrium constant in Equation (12) is taken at the mean effective pressure between the upper and lower limits of x_1 and x_2, corresponding to a small pressure increment, then the term $1/(K_n - 1)$ may be considered a constant, and the resulting integration will lead to the equation

$$x_{n_2} = x_{n_1}\left(\frac{W_2}{W_1}\right)^{(K_n-1)} \quad (13)$$

The summation of this equation results in the following final equation:

$$\sum x_{n_2} = 1 = \sum (x_{n_1})\left(\frac{W_2}{W_1}\right)^{(K_n-1)} \quad (14)$$

When the correct mean effective pressure is obtained, Equation (14) is satisfied. As the resultant liquid is at its bubble point, the bubble-point pressure may be calculated by multiplying the computed x_{n_2} by equilibrium constants of pressures below the mean effective pressure until the following equation is satisfied:

$$\sum x_{n_2} K_n = 1 \quad (15)$$

Equation (14) was solved repeatedly by the use of about 9/10 for the ratio of W_2 to W_1 until a liquid with a bubble point of 14.7 lb./sq. in. abs. at 149°F. resulted. This residual liquid was then cooled to 60°F., and the residual volume was calculated. From residual "liquid" and the liquid volumes calculated at the various reservoir pressures obtained in the repetitive solution of Equation (14), the formation–volume–factor curve in Figure 3 was drawn.

Composite Process. The composite method of determining formation-volume factors and solution gas-oil ratios is similar to the differential method. The liquids obtained at the various pressures in the solution presented above, however, are flashed to 60°F. and 14.7 lb./sq. in. abs. The formation-volume factor is calculated when the volume of liquid at the high pressure and 149°F. is divided by the volume of liquid remaining after it has been flashed to 14.7 lb./sq. in. abs. and 60°F. The moles of gas formed in this flashing process are used in computing the solution gas-oil ratio.

Flash Process. A flash process is one in which the gas evolved from a liquid undergoing a pressure reduction is kept in continuous contact with the liquid. The procedure followed here is to flash the original liquid to various reservoir pressures and then the resulting liquid to stock-tank conditions.

Computed-field-performance Process. This method of obtaining formation-volume factors and solution gas-oil ratios involves the duplication of all the conditions of temperature, pressure, and gas-liquid associations for oil both in the reservoir and on the way to the stock tank. Reservoir volumes of oil at various

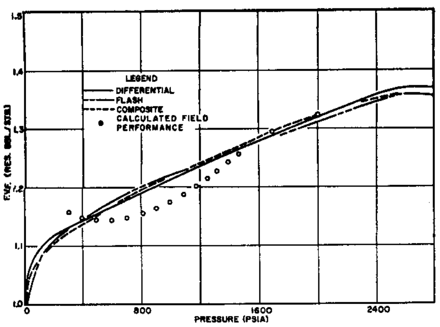

Fig. 7. Calculated formation-volume factors.

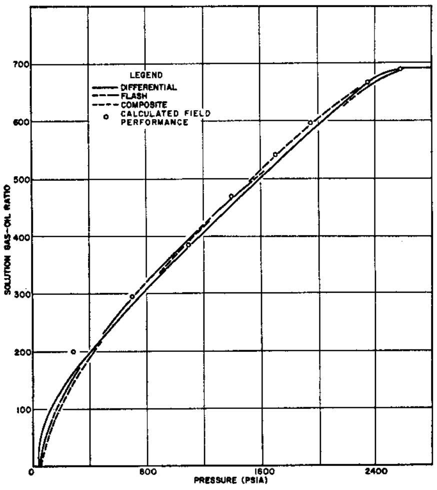

Fig. 8. Calculated solution gas-oil ratios.

pressures may be obtained from the calculations of the performance characteristics of the reservoir. The stock-tank volume of a unit reservoir volume of oil is then obtained by flashing to atmospheric conditions this reservoir oil with any gas flowing with it (the $(\bar{V}_g/\bar{V}_0)_{(i)}$ relationship existing at the specified reservoir pressure). The ratio of these volumes is the formation-volume factor. The solution gas-oil ratio is obtained when the stock-tank volume of oil is divided into the volume of solution gas released. This quantity is the difference between the total volume of vapor released and the volume of vapor initially flowing with the oil from the reservoir, all gas volumes being measured at atmospheric conditions. Incidental to the calculations is the fact that the producing gas-oil ratios may also be found when the total equilibrium volume of gas at the stock tank is divided by the equilibrium volume of oil at the stock tank.

DISCUSSION OF RESULTS

The calculated reservoir pressure and producing gas-oil ratios are shown in Figure 4 as functions of liquid saturation in the reservoir. The curves illustrate the typical characteristics associated with dissolved-gas-drive reservoirs. Figures 5 and 6 show the compositional changes in the gas and liquid phases in the reservoir. The most radical changes in compositions occur between 300 and 14.7 lb./sq. in. abs. Conversely, the first 900 lb./sq. in. drop from the saturation pressure produces only slight compositional changes in both the liquid and the gaseous phases. This latter result is a function of the relative permeabilities of the flowing phases. At 1,700 lb./sq. in. abs. the gas phase begins to flow, producing increasing compositional changes; these changes, however, are not radical but gradual. Initially, the fluid being produced comes entirely from the liquid phase. But when the gas saturation in the reservoir reaches a certain minimum value, the gas becomes mobile and production is then obtained from both phases.

The excess gas being produced with the oil may yield one of two effects: (a) at the lower pressures associated with separators and stock tanks the gas may become undersaturated and vaporize some of the associated liquid or (b) the gas may pass through a retrograde region, especially upon the lowering of temperature associated with production. In this second case the volume of liquid is augmented by the condensing liquid from the gas phase. Both these effects are illustrated in this problem.

Figures 7 and 8 show the formation-volume factors and solution gas-oil ratios calculated by the differential, the flash, the composite, and the phase-behavior–flash processes. Essentially the formation-volume factors for the early stages of pressure reduction check one another. Below 1,700 lb./sq. in. abs., the pressure at which gas begins to flow, condensation from the excess gas being produced with the oil causes a lowering of the formation-volume factor for the computed phase-behavior–flow process below formation-volume factors for the other processes. At the reservoir pressure of 300 lb./sq. in. abs., however, a change in the nature of the excess gas causes liquid vaporization and results in a higher formation-volume factor for the calculated phase-behavior–flow method.

The authors realize that the uncertainties of predicting equilibrium constants, etc., as well as the amount of work required, seriously limit the general application of this method. However, an examination of the formation-volume factors calculated by the various methods will show that serious deviations from actual behavior could result from the use of the type of equations developed by Tarner (*8*) and Muskat (*9*) and the data from the present laboratory procedures. It is worth reiterating also that an ordinary desk-size calculating machine is of no help in making the involved computations that are needed. The method is practicable only where a multielement digital computer of the most recent type is available. The prediction, nevertheless, of the laboratory differential data within 3% indicates that equilibrium constants, etc., do describe the behavior of this particular system. For this reason and for the fact that the deviations noted between the methods can be explained logically, the trends shown are believed to be real. It is hoped that these results will stimulate more effort toward the improvement of present methods of measuring formation-volume factors.

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NOTATION

x = liquid mole fraction
y = vapor mole fraction
C = total moles in reservoir
L = moles of liquid in reservoir
V = moles of vapor in reservoir
M = molecular weight
$\bar{V}_g$ = flowing volume of vapor at reservoir conditions
$\bar{V}_0$ = flowing volume of liquid at reservoir conditions
Z = compressibility factor
R = gas constant
T = absolute temperature
P = absolute pressure
K = vapor-liquid–equilibrium constant
k = relative permeability
μ = viscosity
ρ = density
W = moles of liquid present during differential vaporization

Subscripts

n = component number
t = total
1 = initial condition
2 = final condition
i = instantaneous
0 = liquid
g = gas

LITERATURE CITED

1. Beal, Carlton, *Petroleum Technol.*, T. P. 2018 (March, 1946).
2. Brown, G. G., D. L. Katz, G. G. Oberfell, and R. C. Alden, "Natural Gasoline and the Volatile Hydrocarbons," Natural Gas. Assoc. America, Tulsa, Okla. (1948).
3. Hadden, S. T., *Chem. Eng. Progr.*, **44**, No. 2 135 (February, 1948).
4. Hanson, G. H., M. J. Rzasa, and G. G. Brown, *Ind. Eng. Chem.*, **37** 1216 (1945).
5. Lindsly, B. E., *Petroleum Engineer*, **7**, No. V, 34 (February, 1936).
6. Muskat, Morris, "Physical Principles of Oil Production," 94–95, (1949).
7. Brinkman, F. H., M.S. thesis, Univ. Kansas (1954).
8. Tarner, J., *Oil Weekly*, 114, 32 (June 12, (1944).
9. Muskat, M., *J. Appl. Phys.*, **16**, 147 (1945).